Name: Feedback

SA402 · Dynamic and Stochastic Models

Fall 2023 · Uhan

Quiz 7 – 10/25/2023

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue calculator. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
2	1	
Total		/ 40

Problem 1. Vehicle arrive at the Simplexville Bridge toll plaza between 7 a.m. and 7 p.m. according to a nonstationary Poisson process with integrated rate function

$$\Lambda(\tau) = \begin{cases} 20\tau & \text{if } 0 \leq \tau < 2 \\ 5\tau + 30 & \text{if } 2 \leq \tau < 10 \\ 25\tau - 170 & \text{if } 10 \leq \tau \leq 12 \end{cases}$$

where τ is in hours, $\tau = 0$ corresponds to 7 a.m., and $\tau = 12$ corresponds to 7 p.m.

a. In words, briefly describe the meaning of $\Lambda(4) = 50$ in the context of this problem.

See Problem 3c from the Lesson 11 Exercises for a similar example. Also, see page 1 of Lesson 11 for the definition of the integrated rate function.

Some of you wrote that $\Lambda(4) = 50$ is the number of vehicles to arrive by $\tau = 4$, or 11 a.m. This is close. Will there always be 50 vehicles arriving by $\tau = 4$?

b. What is the probability that the 45th vehicle arrives at the toll plaza at or before 11 a.m.?

To set up the probability statement correctly, consider the following question: if the 45th vehicle arrives at the toll plaza at or before 11 a.m., how many vehicles must have arrived at the toll plaza at 11 a.m.?

See Problem 1e from the Lesson 11 Exercises for a similar problem.

a. If exactly 100 vehicles have arrived by 4 p.m., what is the probability that 135 or fewer vehicles will arrive by 6 p.m.?

See Example 2d in Lesson 11 and Problem 3b from the Lesson 11 Exercises for similar examples.

Problem 2. Vehicles arrive at the nearby Turing Tunnel toll plaza between 7 a.m and 7 p.m according to a nonstationary Poisson process with arrival rate function

$$\lambda(\tau) = \begin{cases} 9 & \text{if } 0 \le \tau < 3 \\ 3 & \text{if } 3 \le \tau < 9 \\ 7 & \text{if } 9 \le \tau \le 12 \end{cases}$$

What is the integrated rate function for this nonstationary Poisson process?

See page 1 and Example 2a in Lesson 11, as well as Problems 1a and 2a from the Lesson 11 Exercises for similar examples.

	$X \sim \text{Poisson}(\mu)$	$X \sim \text{Exponential}(\lambda)$	$X \sim \text{Erlang}(n, \lambda)$
pmf / pdf	$p_X(a) = \begin{cases} \dfrac{e^{-\mu}\mu^a}{a!} & \text{if } a = 0, 1, 2, \dots \\ 0 & \text{o/w} \end{cases}$	$f_X(a) = \begin{cases} \lambda e^{-\lambda a} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$	$f_X(a) = \begin{cases} \dfrac{\lambda(\lambda a)^{n-1} e^{-\lambda a}}{(n-1)!} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$
cdf	$F_X(a) = \displaystyle\sum_{k=0}^{\lfloor a \rfloor} \frac{e^{-\mu}\mu^k}{k!}$	$F_X(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$	$F_X(a) = \begin{cases} 1 - \displaystyle\sum_{k=0}^{n-1} \frac{e^{-\lambda a}(\lambda a)^k}{k!} & \text{if } a \ge 0 \\ 0 & \text{o/w} \end{cases}$
expected value	$E[X] = \mu$	$E[X] = \dfrac{1}{\lambda}$	$E[X] = \dfrac{n}{\lambda}$
variance	$\text{Var}(x) = \mu$	$\text{Var}(X) = \dfrac{1}{\lambda^2}$	$\text{Var}(X) = \dfrac{n}{\lambda^2}$